Solving physics exercises

AIMS

Clarify: to help us understand the problem and picture the physical processes, we need

- a **clear, methodical, structured layout**, starting with
- a **diagram**

We may also sketch, or at least have in mind,

- a **flow diagram** of our strategy for solution.

Solve and test: we need the route to our answer to be reliable and testable. Crucially, the solution must be

- **logical**, so that we can examine every step to check its validity. Remember that physics was historically known as *Natural Philosophy*.
- **complete**, including dimensions (**units** and **vectors**), so that we can see what we're checking
- **honest** – because if we write down something untrue, or which doesn't follow logically, then it's probably wrong

Communicate and convince (ourselves and others) of the physics and our method of solution. We may need to look back on our working; certainly, others will need to be able to follow it. We should therefore

- give a **commentary**: explanatory phrases, stating (in English) the physical laws as they're introduced, and noting explicitly any assumptions (including specific values of constants and parameters, simplifications or limitations to certain regimes of validity)
- indicate key points by underlining the solution, writing '$\Rightarrow$' or '$\therefore$', and noting 'QED', etc.

LAYOUT

Our solutions have four distinct components, which may indeed appear in the order below although they're usually distributed as fragments:

- a **diagram** — this establishes the problem, defines coordinates and dimensions, and forces us to think about what the question means
- **fundamental principles** — physical laws, which are *general* assumptions about the behaviour of the system
- **assumptions** — approximations, specific values and regime limitations, which are *particular* assumptions
- **mathematics** — a series of tautologies, which introduce no new physical information but allow us to view the original physical truths from a different perspective. (An origami swan is still a square piece of paper, although it looks quite different.)

For amusement, we could colour these components differently in our solution: an example is given overleaf.

There are plenty of examples of well-laid-out solutions, in text books, scientific papers and lecture hand-outs. Students should expect to build a library of texts, and should be inspired by photographs of notorious scientists and researchers, almost always taken against a backdrop of books.

TIPS

DO:

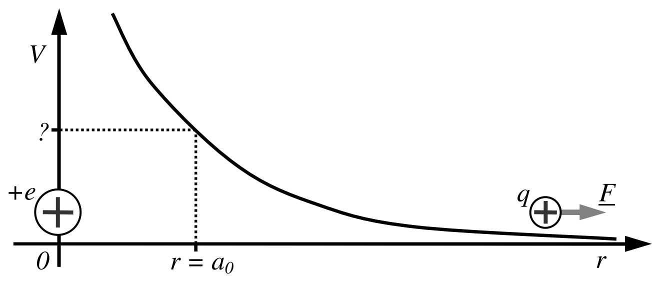
- check logic, validity and dimensions (including vectors)
- work in an algebraic form until as late as possible
- check limiting regimes ($x = 0$, $x \to \infty$ etc)
- sketch graphs of solutions

DON'T:

- introduce equations without explanation: if you don't understand them, they're probably inappropriate
- make big steps: errors are more likely, and they're harder to check
- insert specific numerical values too early

EXERCISE

Calculate the electric potential established by the nucleus of a hydrogen atom at the average distance (a_0 = 5.29x10^{-11} m) of the atom's electron (taking V = 0 at infinite distance).



The force F exerted upon a charge q by a charge +e at a distance r is given by Coulomb's law

$$F = \frac{q\,e}{4\pi\varepsilon_0 r^2}$$

The potential energy of two charges is given by the work done to bring them together, where the work done against a force is equal to the force x distance moved against the force

$$\Delta E = E_2 - E_1 = F\left(-\Delta r\right)$$

The potential energy of our two charges, when separated by a_0, is therefore given by

$$E_{a_0} - E_\infty = -\sum_{r=\infty}^{r=a_0} F\,\Delta r$$

where the force F depends upon the separation r. We must therefore cast this as an integral,

$$E_{a_0} - E_\infty = -\int_\infty^{a_0} F\,\mathrm{d}r$$

which, inserting the particular form of the force from Coulomb's law, gives

$$\begin{aligned}E_{a_0} - E_\infty &= \int_\infty^{a_0} \frac{-q\,e}{4\pi\varepsilon_0 r^2}\mathrm{d}r \\ &= \frac{-q\,e}{4\pi\varepsilon_0}\int_\infty^{a_0} r^{-2}\mathrm{d}r \\ &= \frac{q\,e}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_\infty^{a_0} \\ &= \frac{q\,e}{4\pi\varepsilon_0}\left(\frac{1}{a_0} - \frac{1}{\infty}\right) \\ &= \frac{q\,e}{4\pi\varepsilon_0 a_0}\end{aligned}$$

The electric potential *V* is defined as the electrostatic potential energy per unit charge, ie

$$V = \frac{E}{q}$$

⇒

$$V_{a_0} - V_{\infty} = \frac{e}{4\pi\varepsilon_0 a_0}$$

and we may assume that *V* = 0 at *r* = ∞, so

$$V_{\infty} = 0$$

hence

$$V_{a_0} = \frac{e}{4\pi\varepsilon_0 a_0}$$

Given the specific values $e = 1.60\text{x}10^{-19}$ C
$E_0 = 8.85\text{x}10^{-12}$ F.m^{-1}
$a_0 = 5.29\text{x}1\text{—}^{11}$ m,

we obtain

$$V_{a_0} = \frac{1.6\times10^{-19}}{4\pi\times8.85\times10^{-12}\times5.29\times10^{-11}}\,\frac{\text{C}}{\text{F.m}^{-1}\text{.m}}$$
$$= 27.2\ \text{C.F}^{-1}$$

i.e.

$$V_{a_0} = 27.2\ \text{V}$$

As an example of the importance of identifying assumptions, we may take the following classic proof that $\log_{10}5$ is irrational.

We begin by assuming that $\log_{10}5$ is rational

i.e. we may write $$\log_{10} 5 = \frac{a}{b}$$

where *a* and *b* are integers.

$\Rightarrow$ $$5 = 10^{a/b}$$

$\Rightarrow$ $$5^b = 10^a$$

But any integer power of 5 must end in 5

$$5^b \equiv n \ldots nn5$$

and any integer power of 10 must end in 0

$$10^a \equiv 10 \ldots 000$$

$\Rightarrow$ $$5 = 0$$

This is clearly false, so the initial (only) premise must be false,

i.e. $\log_{10}5$ is irrational.